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Seeger, Billy Frank; Thornton, Joseph Hardin

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A PHASE-SENSITIVE SERVOMECHANISM FOR THE M. I. T. NETWORK ANALYZER

BY

BILLY FRANK SEEGER
JOSEPH HARDIN THORNTON

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By

Billy Frank Seeger, Lieutenant, U. S. Navy
B.S., U. S. Naval Academy 1943

Joseph Hardin Thornton, Jr., Lieutenant U. S. Navy
B.S. Webb Institute of Naval Architecture 1943

Submitted in Partial Fulfillment

of the Requirements for the

Degree of Naval Engineer

From The

Massachusetts Institute of Technology

1950

Cambridge, Massachusetts

May 19, 1950

Professor J. S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Naval Engineer, we submit herewith a thesis entitled, "A Phase-Sensitive Servomechanism for the M. I. T. Network Analyzer."

Respectfully,

Billy F. Seeger
Lieutenant
U. S. Navy

Joseph H. Thornton, Jr.
Lieutenant
U. S. Navy

Acknowledgments

The authors wish to express their appreciation to Mr. Alexander Kusko for his advice and encouragement, and to Mr. D. L. Noiseaux, Mr. L. C. Smith, Mr. W. R. DeHart and Mr. P. E. Smith for their helpful suggestions.

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I. Summary

The object of this thesis was to design a servomechanism to set the phase-angle of the output voltage from a network-analyzer phase-shifter and to maintain this phase-angle during varying load conditions. The phase-angle of the output voltage from the phase-shifter is dependent on its rotor position and the load current. The phase-sensitive servomechanism will maintain the desired phase-angle of the output voltage by continuously controlling the shaft position of the phase-shifter as changes in required angle or load current occur.

The basic design of the servomechanism is illustrated in the block diagram of Figure I. The circuit diagram of the most satisfactory design achieved is shown in Figure II. The units included in this design are as follows:

(1) A phase-sensitive rectifier which functions as the error-sensitive device.

(2) A voltage-amplifier stage to amplify the error signal and to drive the grids of the power amplifier.

(3) A power amplifier to control the field current of the generator in the Ward Leonard drive system.

(4) The Ward Leonard drive comprising a generator and motor of identical ratings and a single-phase induction drive motor.

(5) A synchro-transformer to supply a comparing signal to the error-sensitive device.

(6) A compensation system which was in two sections: a lag network and a negative feedback of a part of the armature terminal voltage.

Since the synchro-transformer used to obtain a comparing signal is essentially a miniature of the phase-shifter used in the network analyzer, a basic assumption of the design is that while the phase-shifter load may vary, the load on the synchro-transformer will remain constant; thus, the phase of the voltage output from the synchro will be a function of the rotor position only.

The original system which was constructed involved the use of a shunt-field-controlled drive motor. Tests on this unit indicated that a change in gear ratio is necessary to obtain a satisfactory torque constant with adequate system stability.

As an interim measure, the shunt-field-control motor was replaced by a Ward Leonard drive system as indicated in Figure II, in order to achieve a stable system with a suitable torque constant. The results of performance tests on this unit indicated the following:

(1) The maximum deviation from the set phase-angle under varying load conditions was 1.3°.

(2) A drift run under constant input and load conditions showed a maximum drift of -0.5° to $+0.2^{\circ}$ over a period of one hour.

Although the design is not sufficiently complete to permit its installation in the Network Analyzer, the above results demonstrate the practicability of the basic design. Recommendations are made in the body of this report which will permit the use of the preferred shunt-field-control drive motor and improve the accuracy to that required by the Network Analyzer.

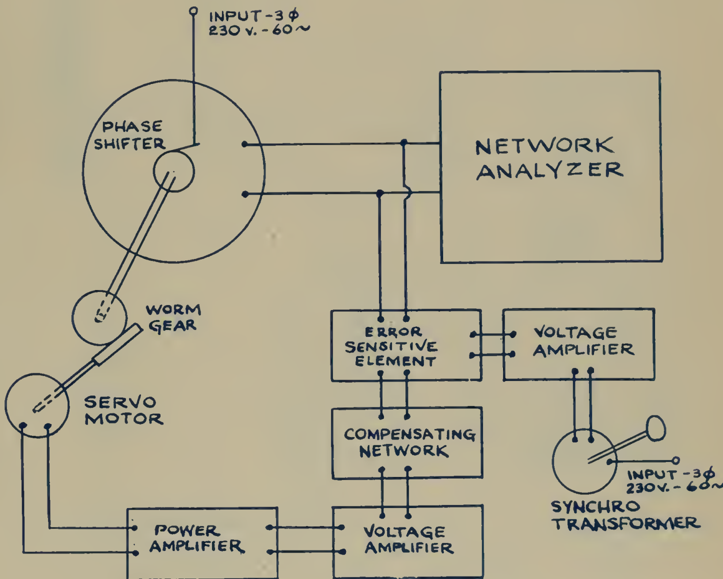

FIGURE I

BLOCK DIAGRAM
of
PHASE - SENSITIVE SERVOMECHANISM

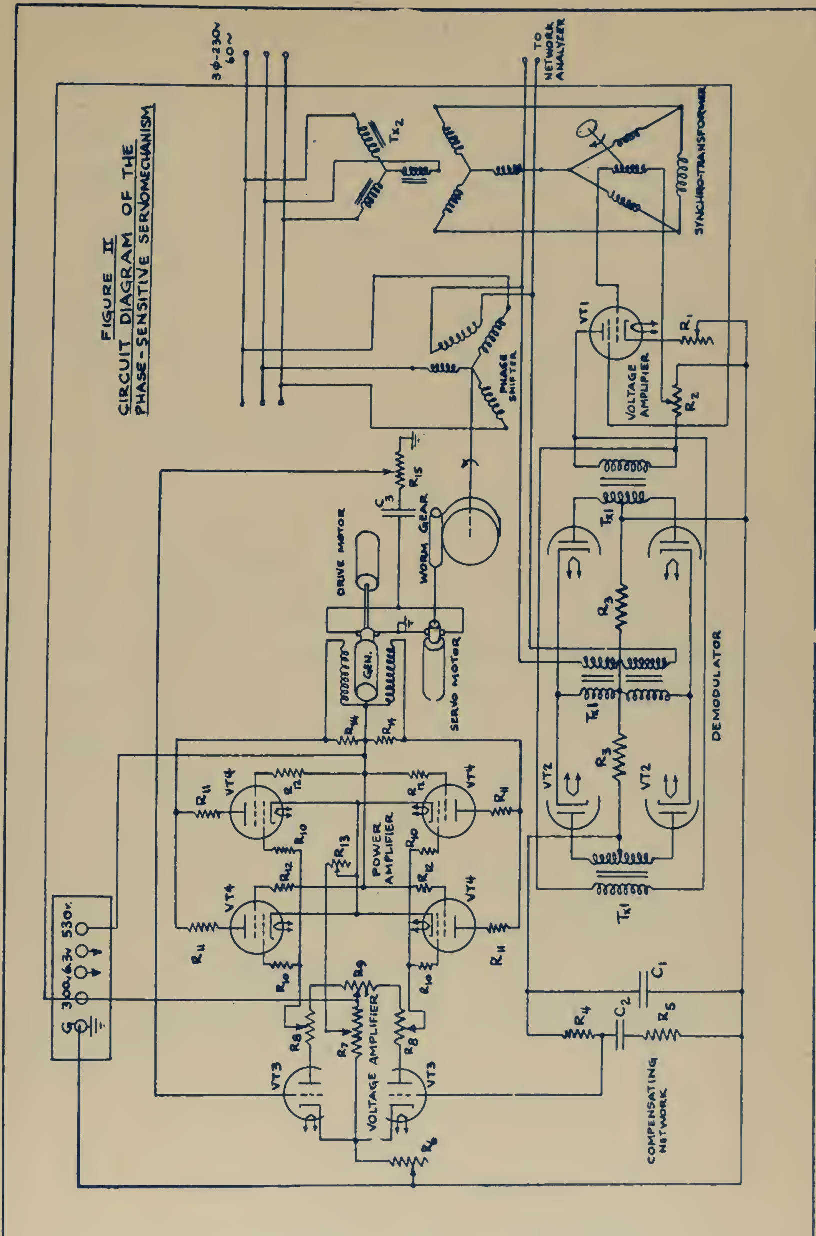

FIGURE II
CIRCUIT DIAGRAM OF THE PHASE-SENSITIVE SERVOMECHANISM

List of Parts for Circuit of Figure II

R_1	2250 ohms	R_9	20,000 ohms
R_2	55000	R_{10}	1000
R_3	75000	R_{11}	10
R_4	3×10^6	R_{12}	100
R_5	440,000	R_{13}	200
R_6	3000	R_{14}	10,000
R_7	5000	R_{15}	2100

C_1 6 uf. filter condenser

C_2 10 uf.

C_3 10 uf.

Tx1 1:1 Transformer

Tx2 2:1 Stepdown Transformer

VT1	6L6 Beam Power Amplifier	VT3	6SL7-GT High Mu Twin Triode
VT2	6H6 Twin diode	VT4	6L6 Beam Power Amplifier

Worm Gear 100:1

Generator: Split field, 110 v. D.C., 0.5 a., 50 watts

Drive Motor: 110 v. A.C.

Servo-motor: 110 v. D.C., 0.5 a., 50 watts

Phase-Shifter: Series conn. Rated 1 amp, 70-410 volts

Parallel conn. Rated 2 amps, 35-205 volts

Synchro-transformer with manual rotor-positioning gearing, 10:1 ratio.

II. Introduction

"The network analyzer consists of an assemblage of resistors, reactors, capacitors, tap-changing auto-transformers and phase-shifters; a system of buses for the interconnection of these elements to represent (on a one-phase line-to-neutral basis) power systems or other electric circuits; and a measuring system for reading currents, voltages, phase-angles, and active and reactive power values within the network....In general any network whose number of elements falls within...the limits of the analyzer... can be represented with a completeness adequate for engineering studies."*

By the representation of complex networks on an analogue basis, the network analyzer permits the solution of problems in electric power transmission, the analytical solutions of which are so tedious as to be impractical. Synchronous machines in the system are represented by several types of phase-shifting transformers.

In general a synchronous machine in a network can be represented with sufficient accuracy by the equivalent circuit of Figure III.

Using the conventional transformer equivalent circuit to represent the particular type of network analyzer phase-shifter for which the servomechanism was designed, the

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* Quoted from Reference (10)

basis for representing the synchronous machines by phase-shifters can be explained.

Fig. III

Equivalent Circuit and Vector Diagram
for the Synchronous Machine

For the phase-shifter (see Fig. IV), the phase of E, the voltage induced in the single-phase stator by the rotating air-gap field of the rotor, will depend on the position of the rotor. The terminal voltage of the phase-shifter, V_2, will however, vary in magnitude and phase with the load current, I_2.

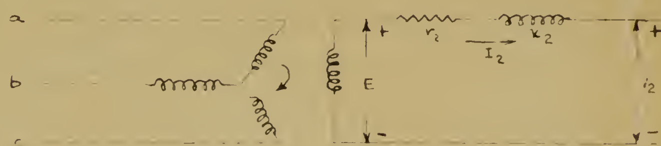

Fig. IV

Transformer Equivalent Circuit for the
Phase-Shifter

With the phase-shifter set for a particular voltage (both in magnitude and phase), the terminal voltage, V_2, can represent the output voltage, V_t, of the synchronous machine of Figure III. If it becomes desirable for stability studies to represent the voltage, E_f, of Figure III, then V_2 can represent this voltage and the synchronous reactance, x_s, can be represented by appropriate static units in the network analyzer.

When one considers that as many as sixteen generating stations may be represented on the analyzer, and that the particular rotor position and voltage tap setting of each phase-shifter is influenced by the setting of all other units, it becomes evident that the adjustment of all phase-shifters to obtain a given set of boundary conditions, will be a tedious trial-and-error process. Thus a primary function of the phase-sensitive servomechanism is to set a particular phase-angle which will remain constant as the other parameters are adjusted. A second and, perhaps, even more important function of the phase-sensitive servomechanism is to extend the scope of the network analyzer to handle studies of transient problems which would require that the boundary conditions be varied continuously in some prescribed manner.

For example, the design of a computer for solving transient stability problems on the Network Analyzer has been developed concurrently with this design.[17]* Assuming a con-

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* Superscripts refer to references as listed in the bibliography.

stant prime-mover power, this computer measures the electrical output of the generator under suddenly applied load conditions and computes the change in phase-angle which will result. The output of this unit, in the form of a shaft rotation, could be used to position the input synchro of the phase-sensitive servomechanism and thus produce a continuous solution to a problem which now can only be handled by a tedious hand calculation and a point-by-point set up of the phase-shifters.

III. Procedure

The preliminary step was the determination of the specifications which the final design should meet. From an investigation of the network analyzer phase-shifters and the manner in which they are used, the following design specifications were established:

(1) The overall accuracy of the completed design should allow less than $\pm 0.5°$ error under extreme conditions of loading. This requirement was based on the present expected accuracy attained by measuring phase-angles with a calibrated phase-shifter.

(2) The system should be free from appreciable drift under steady-state conditions to avoid frequent calibration.

(3) The design must utilize existing phase-shifters in the network analyzer and should be compact and of such a nature as to allow easy and satisfactory incorporation into the network analyzer.

(4) The loading effect of the servomechanism on the phase-shifters should be negligible.

(5) The servomechanism should be stable under all conditions. Since the expected rate of change of input phase was quite small, no specific criterion was established for error under constant-velocity input.

(6) The servomechanism should be relatively insensitive to changes in voltage magnitude.

With these design specifications in mind, the development of the following components was considered:

(1) A device to furnish the required input signal.

(2) A comparing device which would compare the output from the phase-shifter with the input signal and supply an error signal.

(3) Amplifiers to control the drive motor in accordance with the error signal.

(4) A drive motor to position the phase-shifter rotor in accordance with the input signal.

(5) Compensating devices to obtain the required stability.

An investigation of the means at hand for measuring phase-angle was first undertaken. The method utilized at present in the network analyzer is to null the unknown voltage against the output from a calibrated phase-shifter. While this method is highly accurate and furnishes an alternating current signal susceptible to easy amplification, its use with any sort of direct-current drive system appeared impractical, for the following reasons:

(1) Any difference in magnitude of the two voltages compared would appear as an error signal even if the two voltages were in phase.

(2) Distortion of the wave forms would produce an error signal even though the fundamental voltage waves were in phase.

(3) A phase-sensitive circuit would be necessary to ob-

tain a directional-sensitive signal.*

The accuracy obtained by Jacobsen[8] with his phase-angle meter led to a consideration of his circuit as an error-sensitive device. Suffice to say, that considerable modification of his circuit would be required to obtain an error signal. The most straight-forward modification which occurred to the authors involved the use of a phase-sensitive rectifier circuit.

The primary purpose of the phase-measuring device for the particular application intended was to furnish a zero signal for zero error. Therefore, it was decided that a phase-sensitive rectifier, which would in both of the above cases be required to obtain a directional sensitive signal, could be used alone if the proper type of input signal were chosen. If preliminary tests should prove the phase-sensitive rectifier inadequate, its accuracy as an error-measuring means might be improved by some of the techniques used by Jacobsen.

For simplicity of explanation, the half-wave diode phase-sensitive rectifier circuit of Figure V will be utilized.

* A method is described in the Appendix of utilizing an A.C. (two-phase) servomotor which avoids these objections which accompany the D.C. drive. This method suggested itself to the authors only after work had been completed on the D.C. drive system. Hence, due to time limitations, it was not investigated experimentally.

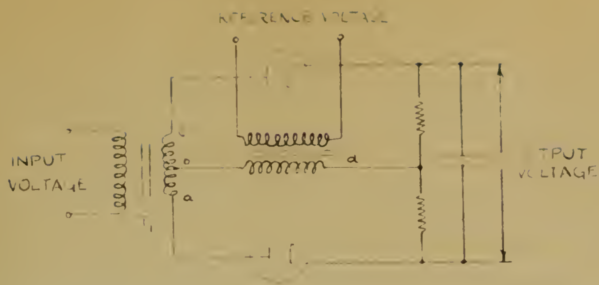


Fig. V.

Assume that the reference voltage above is constant in phase-angle. T_1 is a center-tapped transformer such that voltage, $\overline{ab}$, is equal in magnitude to voltage, $\overline{bc}$. The voltages, V_1 and V_2, applied to the two rectifiers will then be equal in magnitude as shown in the solid-line vector diagram of Figure VI, if the voltage $\overline{ac}$ is 90° out of phase with the reference voltage. With these two equal voltages,

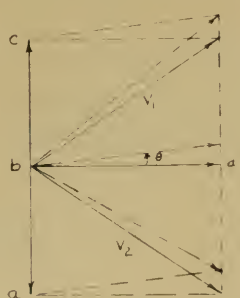


Fig. VI

V_1 and V_2, applied to the rectifier portion of the circuit, the D.C. output voltages will be zero for input voltages 90° out of phase, regardless of their respective magnitudes.

Assuming, for the moment, that the voltages, $\overline{ab}$, $\overline{bc}$, and $\overline{bd}$, are constant in magnitude, the variation of voltages, V_1 and V_2, as the phase angle departs from 90° by a small angle, θ, can be observed. Referring to the dashed-line vectors, Figure VI

$$|V_1| = |\overline{bd}\cos\theta + j\overline{bc} + j\overline{bd}\sin\theta| \tag{1}$$

$$|V_2| = |\overline{bd}\cos\theta + j\overline{ba} - j\overline{bd}\sin\theta| \tag{2}$$

$$\text{Since} \quad |\overline{ba}| = |\overline{bc}|$$

$$|V_1| = \sqrt{\overline{bd}^2 + \overline{bc}^2 + 2\overline{bc}\cdot\overline{bd}\sin\theta}$$

$$= \sqrt{\left(1 + \frac{2\overline{bc}\cdot\overline{bd}}{\overline{bd}^2 + \overline{bc}^2}\sin\theta\right)\left(\overline{bd}^2 + \overline{bc}^2\right)}$$

$$|V_2| = \sqrt{\left(1 - \frac{2\overline{bc}\cdot\overline{bd}}{\overline{bd}^2 + \overline{bc}^2}\sin\theta\right)\left(\overline{bd}^2 + \overline{bc}^2\right)}$$

For small angles:

$$|V_1| - |V_2| \approx \left[1 + \frac{2\overline{bc}.\overline{bd}}{\overline{bd}^2 + \overline{bc}^2}\cdot\frac{\theta}{2} - 1 + \frac{2\overline{bc}\cdot\overline{bd}}{\overline{bd}^2 + \overline{bc}^2}\cdot\frac{\theta}{2}\right]\sqrt{\overline{bd}^2 + \overline{bc}^2}$$

$$|V_1| - |V_2| = \frac{2\overline{bc}.\overline{bd}.\theta}{\sqrt{\overline{bd}^2 + \overline{bc}^2}} \tag{3}$$

From Equation (3) it can be seen that:

(1) The D.C. output voltage, which is proportional to $|V_1| - |V_2|$, is proportional to θ for small departures from the 90° phase position.

(2) The magnitude of the output voltage per degree departure from 90° is limited principally by the smaller of the two voltages, $\overline{bc}$ or $\overline{bd}$, if their magnitudes differ appreciably.

In order to utilize such a sense-detecting rectifier as an error-measuring device, the input must consist of a voltage of the same frequency as the reference; it must also be easily varied in phase. A simple means of obtaining this is through the use of a synchro-transformer with three-phase excitation. If the load is kept constant, the phase-angle of the output voltage is a function of the rotor position only. To insure a constant load, a buffer amplifier was inserted between the synchro-transformer and the sense-detecting rectifier as shown in Figure VII.

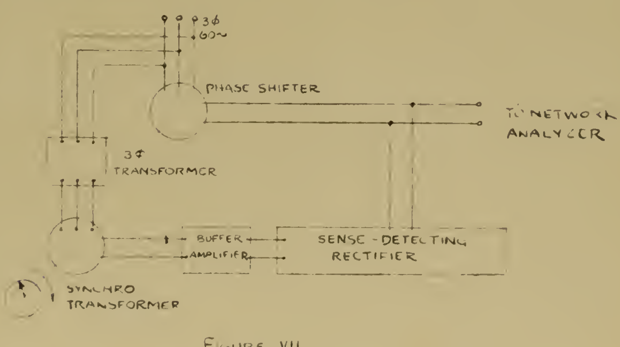


FIGURE VII

PLATE I

Synchro-Transformer with

Rotor-Positioning System

The input synchro rotor-positioning dial indicator is calibrated in such a manner that the synchro output voltage is 90° from the dial setting.

Of the several types of sense-detecting rectifiers similar in principle to the half-wave diode circuit described above, the full-wave diode circuit was decided upon for the following reasons:

(1) The use of diodes reduces the possibility of drift because of their more stable characteristics.

(2) The full-wave rectifier produces less ripple for a given filter circuit. The ripple voltage must be reduced to the point that A.C. saturation of the D.C. amplifiers (which amplify the error signal) does not occur.

The circuit diagram for the full-wave diode sense-detecting rectifier appears in Figure VIII.

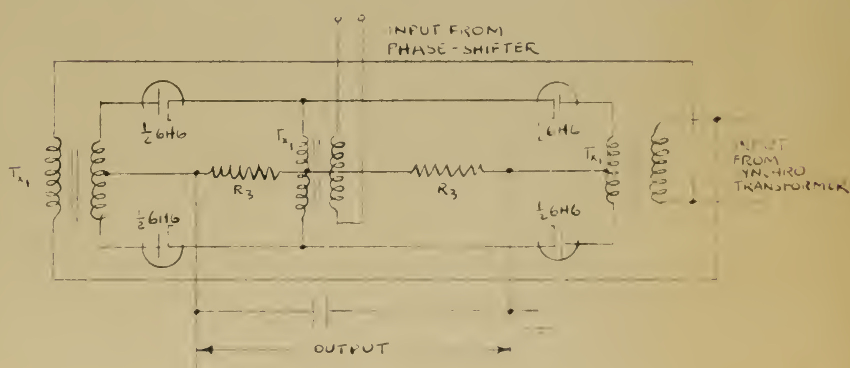


FIGURE VIII

Having selected the error-sensitive element, the next step was the selection of a servo-drive system. Work done by Mayer and Clodt in their thesis[2] indicated that the torque of the phase-shifter underwent a variation with rotor position which was unpredictable. The greatest torque that they measured was about 2 ft.lbs. and this figure was used in preliminary estimates of steady-state error. Of greater value in choosing the motor to be used was the fact that they had used a 1/50 H.P. motor with a 60:1 gear reduction to position one of the network analyzer phase-shifters with satisfactory results.

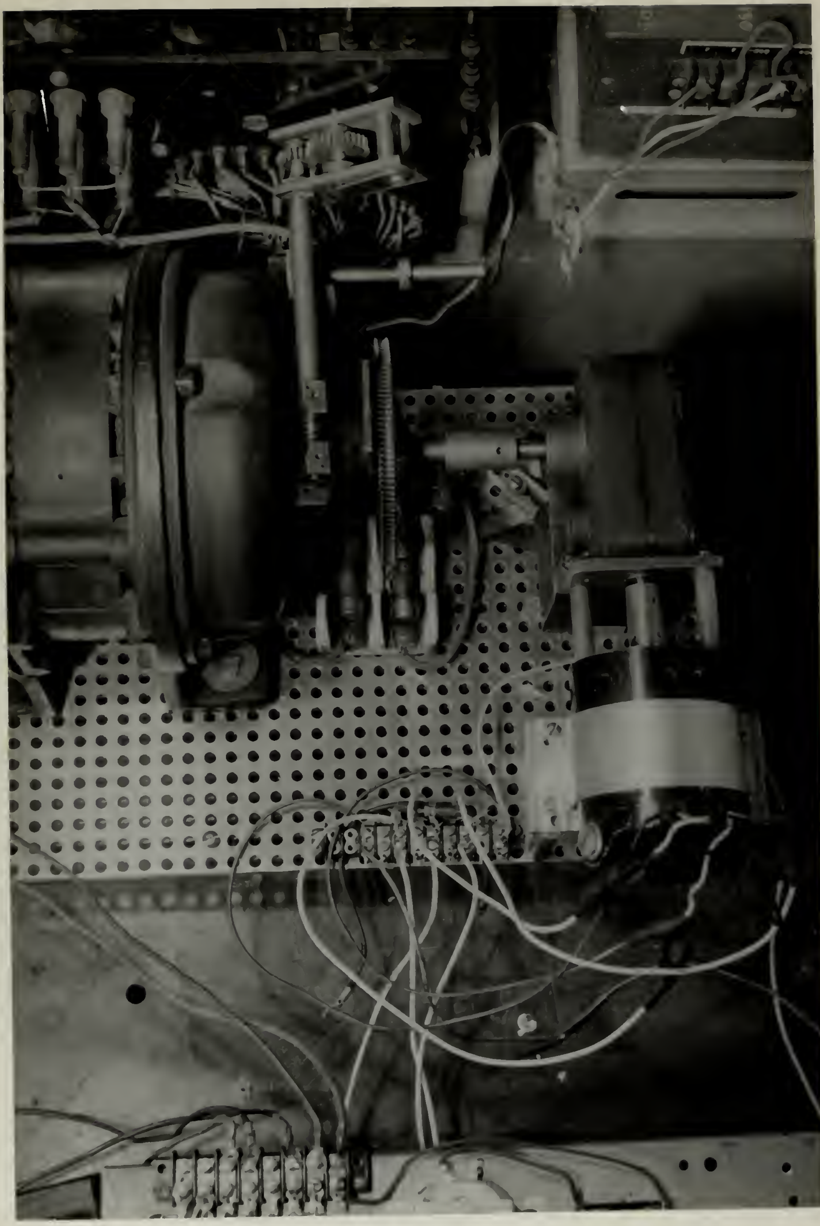
The desirability of compactness indicated the use of a D.C. motor with shunt-field control or a D.C. motor with armature control. The shunt-field-control motor was chosen because relatively low power tubes could be used in the power amplifier. In addition, while shunt-field control can be provided by a simple push-pull amplifier, the use of a bridge circuit is indicated to obtain armature control and the problems of direct current stability of the amplifiers are greater.

It was considered that the advantage of a more stable amplifier system more than offset the disadvantage that a separate constant current source would be required for the shunt-field-controlled motor.

In view of the above discussion, it was decided that the 50-watt shunt-field motors available in the Servomechanisms Laboratory would be satisfactory for experimentation. The remainder of the design was predicated on the use of one of these

PLATE II

Phase-Shifter, Worm-gear and Drive-motor



motors with a gear reduction of 100:1.

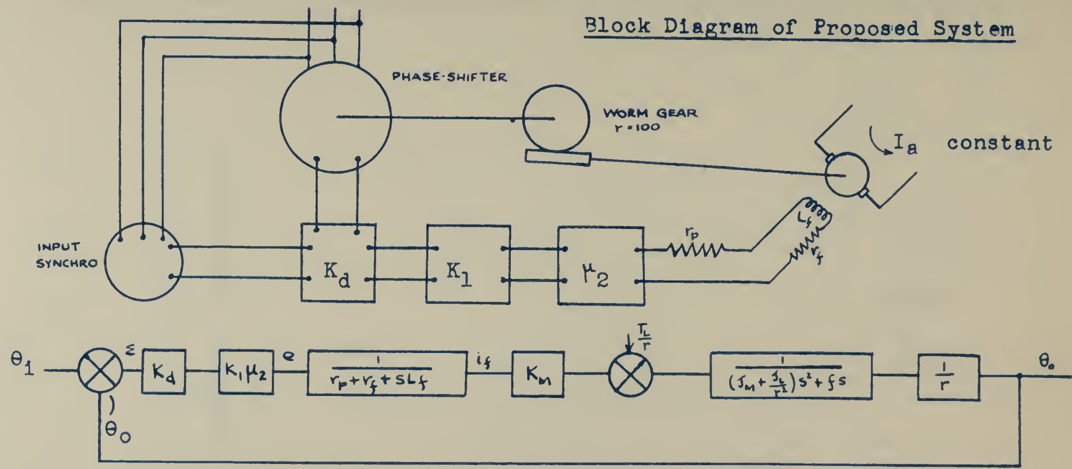
At this point it became possible to construct a block diagram of the proposed system which aided in further analysis. (See Figure IA.)

Calculations (see Sample Calculations, Appendix) indicated that at least two stages of D.C. amplification would be required following the phase-sensitive rectifier in order to obtain the desired torque constant for the system. For the voltage amplifier, a 6SL7 tube was chosen and arranged in push-pull with one grid grounded as shown in Figure II. The expected gain for this stage was about 40 volts per volt.

For the power amplifier, one of the power amplifiers used in the Servomechanisms Laboratory was used after some slight modifications had been made to adapt it to the circuit. In this amplifier two 6L6 tubes in parallel supply each half of the motor field current in a push-pull arrangement. For quiescent conditions, with the current in each field of the same magnitude, the flux of one field is opposed by the flux of the other field, giving a net field flux of zero. When an error signal is applied, the current in one field increases, and that in the other, decreases. Thus is applied a net control flux proportional to the error-signal both in magnitude and direction.

In the provision of a constant current source for the armature, advantage was taken of the tube characteristics of the type 6L6 beam power amplifier tube. The relatively large current requirements of the armature led to the use of six 6L6

Block Diagram of Proposed System



Key

K_d = gain in phase-sensitive rectifier, volts per degree
K_1 = voltage amplifier gain volts/volt
μ_2 = amplification factor, power amplifier
r_f = motor field resistance
L_f = field inductance
K_m = motor torque constant
J_m = motor armature inertia
J_L = phase-shifter and associated gear inertia
f = viscous friction coefficient

Figure IX

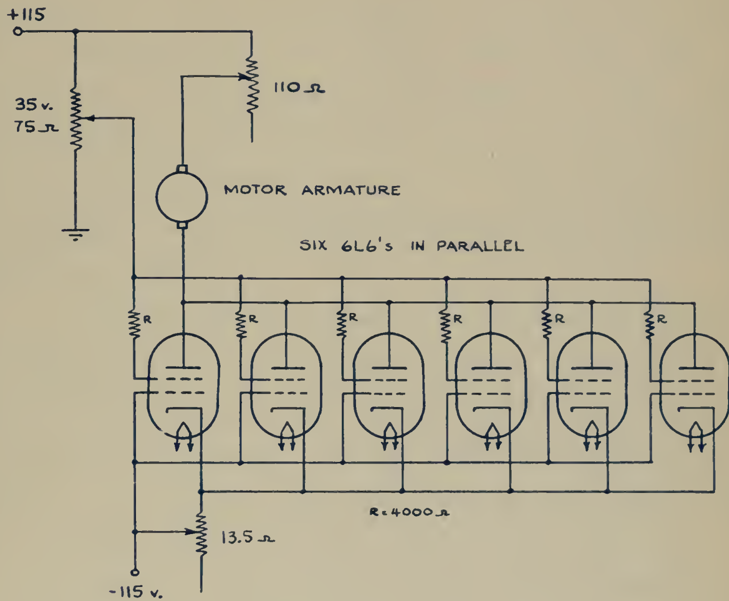


FIGURE X

CONSTANT CURRENT SOURCE TO SUPPLY MOTOR ARMATURE CURRENT

tubes in parallel as shown in Figure X. (See also Sample Calculations in Appendix.)

Upon conclusion of the preliminary calculations, construction of the various components was begun. To permit ease of alteration and to save time a "bread-board" type of construction was used for most of the elements. Only qualitative performance tests were made on the elements before the loop was closed since it was expected that changes would be necessary as more knowledge was gained of the system performance as a unit.

When the loop was closed, the system was unstable, oscillating at about 100 cycles per minute. A variac was inserted in place of the input synchro-transformer to serve as a variable gain control, and the gain was reduced until stability was attained. When stabilized by this method, however, the torque constant was so low that the system was relatively useless and it became evident that compensation would be required. An investigation of the gain actually being developed by the D.C. amplifiers disclosed that only about 10 volts per volt was being obtained although the circuit had been designed to give a gain of about 200. After checking the amplifiers to determine the source of trouble, it was found that the ripple component of the output from the phase-sensitive rectifier was excessive and was causing saturation of the D.C. amplifiers. The excessive ripple, coupled with the high quiescent potential used in the motor field coils also caused the field coils to arc to ground.

The following corrective measures were taken:

(1) The capacitance of the filter condenser was increased to reduce the ripple.

(2) The D.C. motor field was rewound using a better type of insulation.

After these steps had been taken, an investigation of possible methods of compensating the system was begun. While a frequency response test would have been the most straight-forward method (see Appendix for method), the construction of additional equipment would have been required for which time was not available. In addition certain improvements suggested themselves which should be accomplished before a frequency response test is made.

From Figure II:

For $T_L = 0$

$$KG = \frac{\theta_o}{\varepsilon} = \frac{K_d K_1 \mu_2 K_m}{r}\left(\frac{1}{r_p + r_f + sL_F}\right)\left(\frac{1}{(J_m + \frac{J_L}{r^2})s^2 + fs}\right)$$

$$= \frac{K_d K_1 \mu_2 K_m}{fr(r_p + r_f)} \cdot \frac{1}{s(\tau_1 s + 1)(\tau_2 s + 1)} \qquad (4)$$

$$\text{where } \tau_1 = \frac{L_F}{r_p + r_f} \qquad \tau_2 = \frac{J_m + J_L/r^2}{f}$$

For $\theta_i = 0$

$$\frac{T_L}{\theta_o} = \frac{T_L}{\varepsilon} = \frac{1}{(J_m + \frac{J_L}{r^2})s^2 + fs} + \frac{K_d K_1 \mu_2 K_m r}{r_p + r_f + sL_F}$$

$$\frac{T_L}{\varepsilon_{ss}} = K_t = \frac{K_d K_1 \mu_2 K_m r}{r_p + r_f} \qquad (5)$$

When the gain of the shunt-field-control system was reduced until the system was satisfactorily stable, the gain through the demodulator and amplifiers, represented in Equation (4) by $\frac{K_d K_1 \mu_2}{r_p + r_f}$ was approximately 2.0 milliamps per degree of error. (See plot, Figure XII) Substituting this value in Equation (5), together with the motor torque constant $K_m = 77$ in.oz. per ampere and the gear ratio, $r = 100$ and solving for K_t, gives $K_t = 2.0(100)77/1000 = 14.4$ in.oz. per degree.

By taking a frequency response test on the system and inserting a lag network, it may be possible to increase the allowable gain by a maximum factor of ten, and thus increase the torque constant. An increase in torque constant to about 144 in. oz. per degree would still result in a torque error of over 2 degrees for the assumed load torque of 384 in. oz. It was concluded, therefore, that adequate compensation of the existing system was not feasible using a passive network of this type.

A more promising method of improving the performance of the system is to increase the gear ratio. From Equation (4) it can be seen that increasing the gear ratio by a factor of ten will allow an increase of ten in the amplifier gain without changing the overall gain of the system. This would increase the torque constant by a factor of 100 as indicated by Equation (5).

In addition the load inertia referred to the motor would be decreased by a factor of 100; the friction in the system would be increased by an unpredictable amount; and additional backlash would probably be introduced into the gear train. While these last factors make the exact prediction of the system performance uncertain when the gear ratio is increased from 100 to 1000, it is believed that the major purpose--to obtain a satisfactory torque constant with suitable stability--can thus be attained. Frequency response studies after the introduction of the new gear ratio may indicate further improvements which could be obtained with passive networks, but it is considered that the change in gear ratio should be made prior to any frequency response test.

Time considerations prevented the accomplishment of this improvement prior to the submission of this report. As an interim measure, the torque constant was increased by substituting a Ward Leonard drive system for the shunt-field-control motor, as shown in the wiring diagram, Figure II. By compensating this system with a lag network and armature feedback (see Figure II), the torque constant obtained with the Ward Leonard drive yielded a maximum torque error, under full load, of about 0.5 degree.

Experimental work was concluded by testing the ability of the servomechanism with the Ward Leonard drive to hold a set-phase-angle under varying load conditions of the phase-shifter. The results of these tests are presented in the Summary of Results.

IV. Results

The results of the investigation are submitted in two categories, as follows:

(1) The results of performance tests upon the system when the Ward Leonard drive was used. These indicate the practicability of the phase-sensitive servomechanism.

(2) The results obtained using the shunt-field-controlled drive motor system. These indicate the changes in the design necessary to accommodate this type of drive which is simpler and hence more suitable for use with the Network Analyzer.

(1) Results Using Ward Leonard

(a) From the data shown in Table I, the maximum departure from the set-phase-angle which results from changes in load from 0.4 to 1.0 ampere, is $\pm 1.3^{\circ}$.

(b) Of this error, approximately 0.5 degree can be ascribed to load torque and static friction in the drive system. (See Figure XI.)

(c) The remaining error, about 0.8 degree, can be attributed to the error-measuring device.

(d) Under steady input conditions, the drift in phase-angle observed over a period of an hour ranged in value between 0.2° and -0.5°.

(2) <u>Results Using Shunt-Field Control System</u>

The measured gain through the demodulator and amplifiers for the uncompensated shunt-field-control system was 2.0 milliamps per degree of error. This gives a calculated torque constant of 14.4 in. oz. per degree of error.

Table I

Phase-Angle Set for Load of 0.4 amps.	Measured departure from the set-phase-angle as load was changed to the indicated values. Voltage of phase-shifter constant 75v.		
	0.6 amp. load	0.8 amp. load	1.0 amp. load
35.7°	-0.5°	-0.5°	-0.9°
107.2	+0.3	+0.1	0.0
179.5	-0.5	-0.6	+0.4
215.8	-0.9	-1.3	-0.6
323.9	+0.3	-0.1	+0.4

Note: All phase-angles measured with the Network Analyzer calibrated phase-shifter.

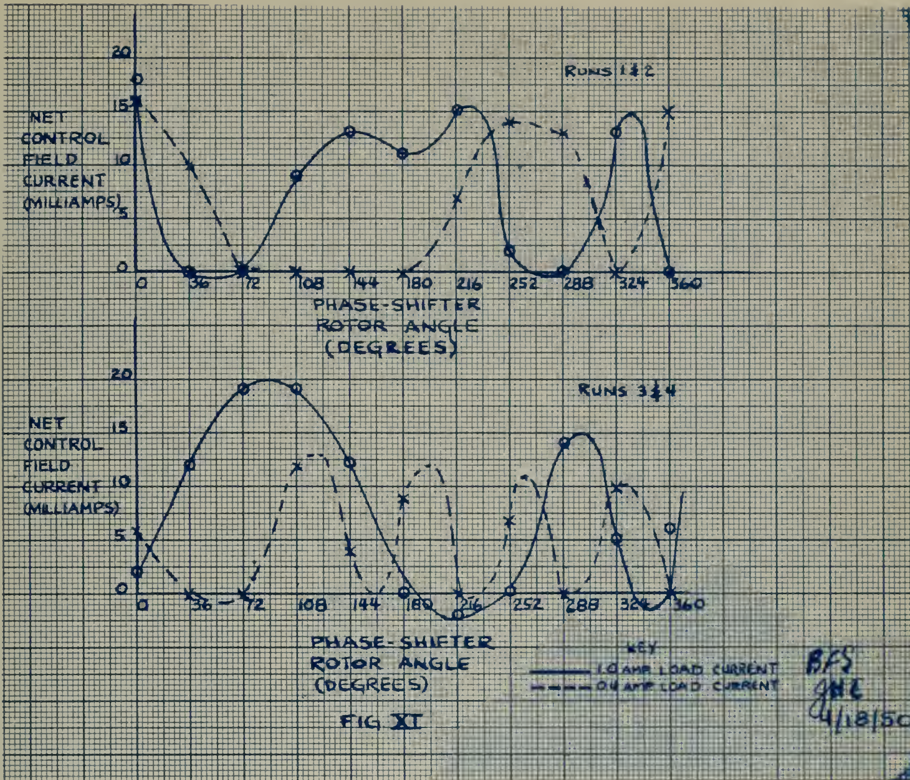
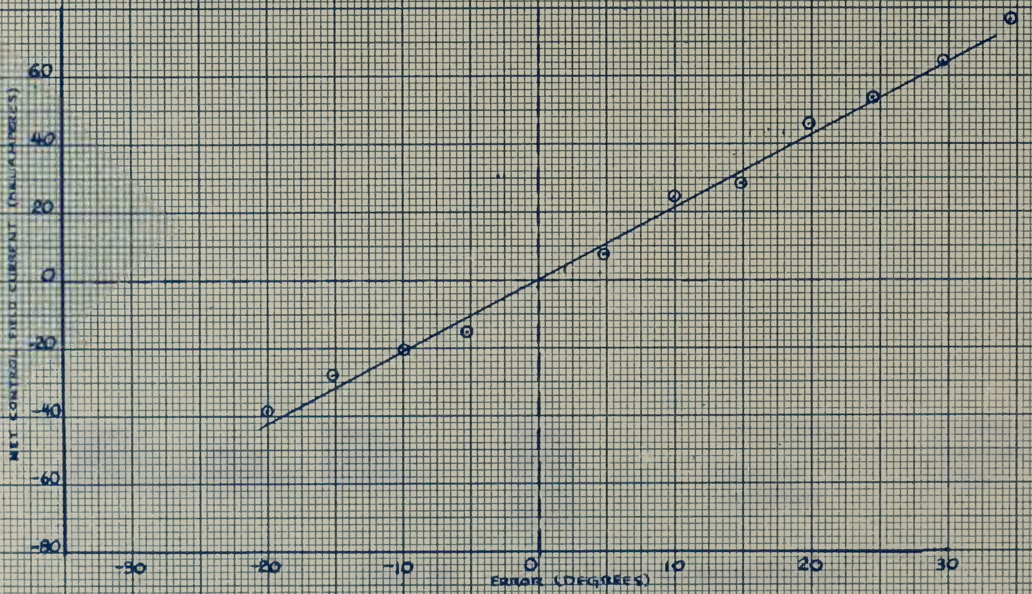


FIG XI

The generator field current of the Ward Leonard system was measured at 36°-intervals as the input synchro was rotated to position the phase-shifter throughout the 360°-range. The measured gain used during these runs was 40 milliamps per degree error. Although the results plotted above show an unpredictable variation indicating the presence of static friction, the maximum field current observed in a number of runs was 20 milliamperes which corresponds to a torque error of 0.5°.



Measurement of Allowable Gain for Stable Shunt-Field Control System

Control Field Current vs. Phase Error

Figure XII

BAS
JWC
4/15/50

PLATE III GENERAL VIEW OF SERVOMECHANISM

V. Discussion of Results

As stated in the Procedure, the use of the Ward Leonard Drive was a matter of expediency. The only purpose of this modification in the design was to obtain an evaluation of other system components. From this viewpoint, the use of this system was successful in that it allowed an investigation of the accuracy of the error-measuring system under similar conditions to those expected to be encountered by the completed design. By measuring the motor field current under load conditions and the gain through the error channel, it was possible to determine approximately how much of the measured error was due to the load torque and to static friction in the system. The remaining error of about 0.8° could only be ascribed to the error-measuring system.

Further measurements on the phase-sensitive rectifier disclosed that the transformers were not accurately center-tapped. It can be shown (see Appendix) that this will introduce an error in the circuit which is directly proportional to the magnitude of the error in positioning the center-tap. Any unbalance of the diode unit will contribute a similar error. If the magnitudes of the input voltages are held constant, this error can be corrected by calibration of the input synchro; but recalibration will be required for each change in magnitude of the input voltage. An accurately balanced phase-sensitive rectifier is required, therefore, for optimum performance of this unit.

The effects of waveform distortion of the input voltages upon the accuracy of the circuit are not predictable. Further experimentation may disclose that some preliminary shaping of the input voltage waves, as was done by Jacobsen[8], will be necessary in order to obtain the required accuracy; but it seems advisable to avoid this measure if possible, by first improving the performance of the existing unit in the manner specified above.

The results obtained with the shunt-field-drive motor demonstrate the necessity for increasing the gear ratio to obtain satisfactory performance of this unit. While the introduction of electrical damping may present an alternative method of achieving the same result, it is believed that due to the low velocity requirements of the system, the change in gear ratio is a more desirable solution.

Since the input voltage to the phase-sensitive rectifier is limited by the maximum voltage (120 v.) which can be applied to the diodes, provision must be made for a voltage reduction from the phase-shifter. To minimize the load which the servomechanism places on the phase-shifter, it appears desirable to accomplish this voltage reduction through the use of a large resistor (about 0.1 megohm) across the phase-shifter output. By tapping this resistor at various points, and using this voltage to drive a buffer amplifier, provision can be made for large changes in magnitude of the phase-shifter voltages. As shown in the analysis of this unit, it should, if properly balanced, be insensitive to changes in

voltage magnitude within the limits of the diodes.

Although an attempt was made to calibrate the input synchro-transformer, the results were unsatisfactory. When the input dial was rotated through 360° and the output voltage phase-angle was measured with the Network Analyzer calibrated phase-shifter, errors as large as 4.5° were observed. This may be attributed to errors in the synchro-transformer itself and possibly, to the back-lash in the gearing system.

VI. Recommendations

The following is a summary of the recommendations for improvements in the design of the phase-sensitive servomechanism:

(1) Change the gear ratio in the shunt-field-control drive from 100 to 1000.

(2) Use a more stable tube such as the 6SJ7 in the voltage amplifier. The change in gear ratio will reduce the D.C. amplification necessary to obtain a satisfactory torque constant, and permit the use of a tube with lower gain.

(3) Improve the performance of the phase-sensitive rectifier by procurement of more carefully selected components.

(4) Construct a circuit to provide for major changes in magnitude of phase-shifter voltage.

(5) Improve input synchro system including gearing, to allow final calibration.

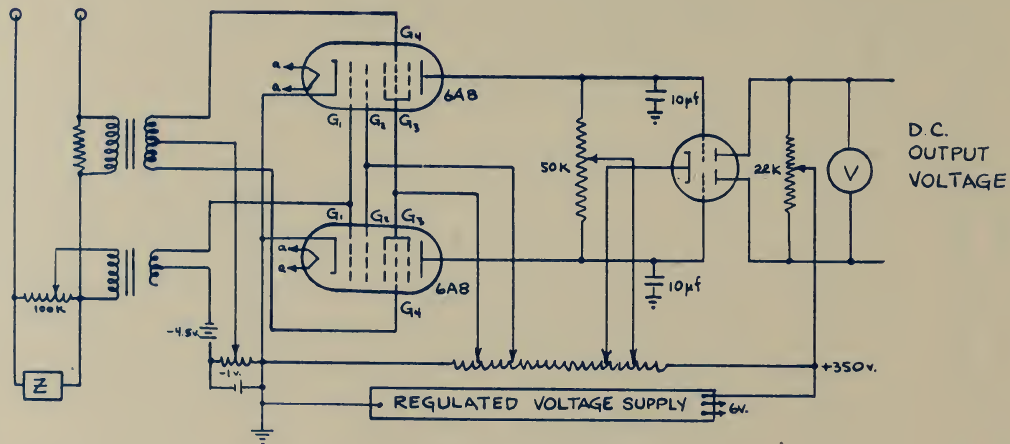
In addition it is recommended that the two-phase drive motor system outlined in the Appendix be investigated further.

VII. Appendix

A. Investigation of Electronic Wattmeter Circuits

The original aim of this thesis was the continuation of the development, started by Mayer and Gloet[2], of a power-sensitive servomechanism for the network analyzer. Problems are frequently encountered where it is desirable to hold the power output of a particular phase-shifter at a constant value, and as in the case of the phase-sensitive servomechanism, the power-sensitive servomechanism would result in a reduction of time required to set up this type problem. The authors' work on this problem consisted of the investigation of two electronic circuits which showed promise of serving as power-sensitive elements. From the results of this investigation, it was concluded that neither of these two circuits was sufficiently accurate and stable to serve the purpose intended. The most straight-forward method of obtaining a power-sensitive element is therefore considered to be the adaptation of one of the standard dynamometer-type wattmeters or watt-hour meters to produce a voltage signal proportional to power.

The time available for development of such a power-sensitive device was inadequate and the authors instead turned their efforts to the development of the closely allied phase-sensitive servomechanism. As a possible aid to future investigators the following brief summary of the experimental work on the electronic wattmeter circuits is presented.



PIERCE ELECTRONIC WATTMETER CIRCUIT
(AS MODIFIED BY MAYER & GLODT)

FIGURE XIII

The first circuit investigated was developed by Pierce[1]. The Pierce circuit as modified by Mayer and Glodt[2] is shown in Figure XIII. The short time available to Mayer and Glodt prevented their obtaining conclusive results with the circuit and the advantages of an electronic wattmeter element in the servomechanism application prompted further experimentation.

Pierce's use of a multielectrode tube as a wattmeter can be explained by the use of Figure XIV (a, b, and c).

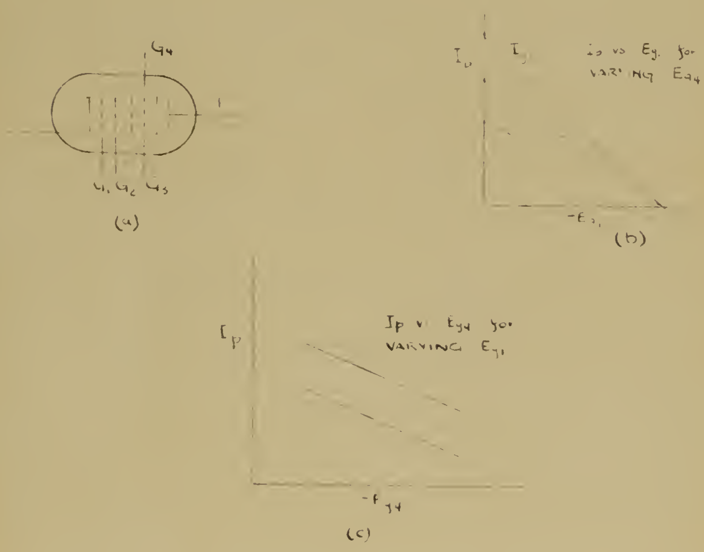


FIGURE XIV

In Figure XIV (a), G_2 and G_3 are held at positive fixed potentials above the cathode. G_1 and G_4 are biased with a negative potential with respect to the cathode. In such a case, the current passing through G_1, I_{g1}, is dependent only on the potential of G_1, being independent of the potential of G_4. The proportion of I_{g1} reaching the plate is, however, dependent only on the potential of G_4.

For these conditions, $I_p = I_{g1} \cdot \Phi(E_{g4})$.

If I_p as a function of E_{g1}, with E_{g4} fixed at various values, can be represented by a series of straight lines terminating at a point as shown in Figure XIV (b)

then, $I_{g1} = K(G_1 - A)$

If, also, I_p as a function of E_{g4} for some value of E_{g1} in this region, is a straight line, then

$$\Phi(E_{g4}) = K_2(G_4 - A_2).$$

I_p will then have the form,

$$I_p = AE_{g1} + BE_{g1}E_{g4} + CE_{g4} + D .$$

The terms, AE_{g1} and CE_{g4} are of an alternating current type. The direct current of the product term will, therefore, be proportional to the power if the grid signals are made proportional to the voltage and current to the load, as is done in the circuit, Figure XIII. The above explanation is paraphrased from Pierce's paper[1] and the reader is referred to this paper for further information and details. The 6A8 tubes used by Mayer and Glodt[2] are a replacement type for the 2A7 tubes originally used by Pierce.

Preliminary tests on the circuit of Figure XIII yielded the following results:

(1) The circuit response was linear under varying conditions of voltage and current, if the power factor of the load was held constant, as shown in Figure XV.

(2) The circuit response to variable power-factor loads is shown in Figure XVI. If the voltages and current were held constant and the power factor of the load varied, a linear variation was obtained but the line did not pass through the origin. See Curve 1, Figure XVI. That is, a zero power indication would not have resulted for a zero power-factor load. Pierce's results[1] indicated that his original circuit had met this test.

When the load power was held constant and the power factor varied, the results were as indicated by curves 2 and 3 of Figure XVI for 50- and 30-watt loads, respectively: Curve 3 is included since it was the only run taken which included a lagging power-factor load. Curve 2, Figure XVI, shows that the response to a constant-power, variable-power-factor, load is nearly linear. Curve 3, Figure XVI, shows that the slope of the "line" continues in the same direction for leading and lagging power factors.

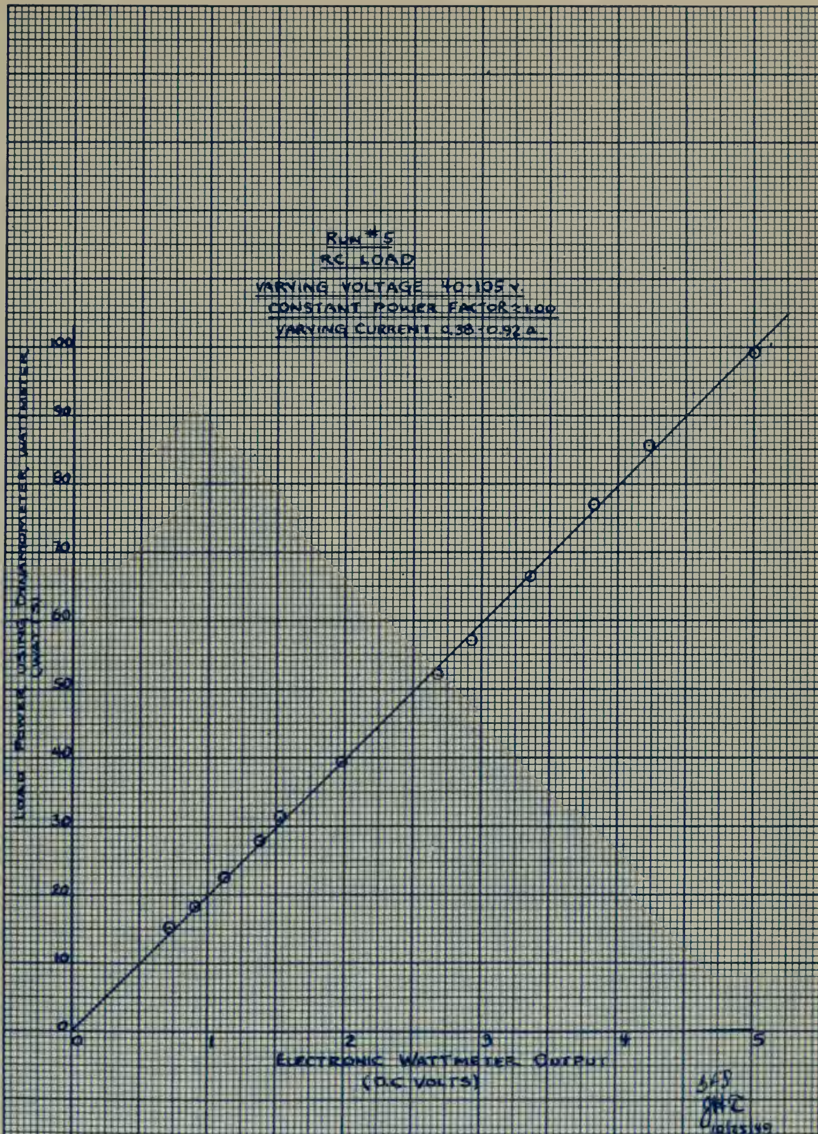

RUN #5
RC LOAD
VARYING VOLTAGE 40-105 V.
CONSTANT POWER FACTOR = 1.00
VARYING CURRENT 0.38-0.92 A
LOAD POWER USING DYNAMOMETER WATTMETER (WATTS)
0
10
20
30
40
50
60
70
80
90
100
0
1
2
3
4
5
ELECTRONIC WATTMETER OUTPUT
(D.C VOLTS)

FIGURE XV

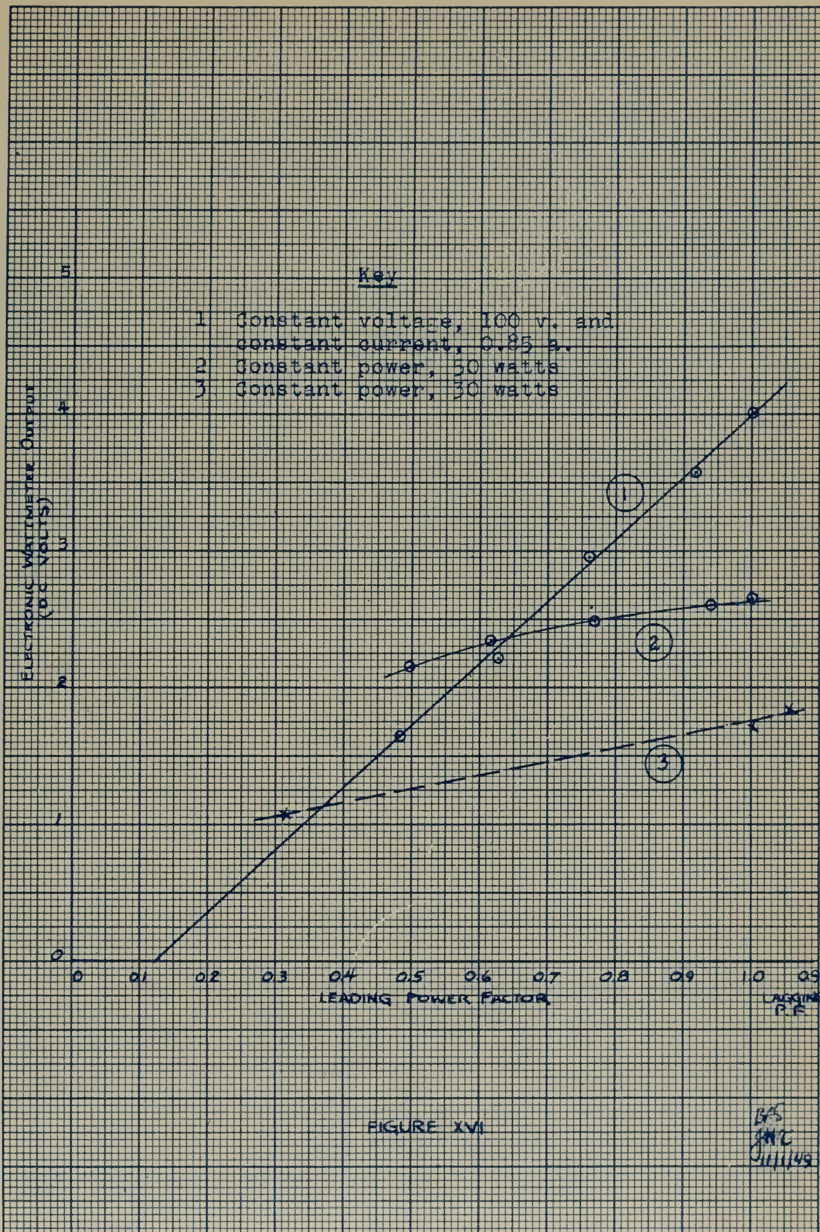

Key
1 Constant voltage, 100 v. and
constant current, 0.85 a.
2 Constant power, 50 watts
3 Constant power, 30 watts
ELECTRONIC WATTMETER OUTPUT (DC VOLTS)
LEADING POWER FACTOR
LAGGING P.F.
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.9
1
2
3
4
5
BS
JHC
11/1/49

FIGURE XVI

If it is assumed that the phase-shift in each transformer is not the same these results can be explained. It is expected that if curve 3, Figure XVI were extended into the region of very low power factors, the slope of the curve would reverse at some point where the actual inputs to the circuit were in phase. It was therefore deduced that if a phase-shifting circuit were introduced into one of the inputs in such a manner that the input signals would actually be in phase for a unity power-factor load, the circuit response would be improved.

Accordingly, the circuit was modified to place a variable capacitance in parallel with the transformer which received the signal proportional to voltage as shown in Figure XVII.

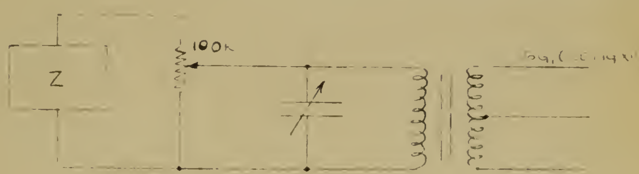


Fig. XVII

The results of this change are shown in the plots of Figure XVIII. The constant-voltage, constant-current run, curve 1, passes through the origin as required. The constant-power run curve 2, is more nearly parallel to the horizontal axis. By variation of the phase-shift introduced, the slope

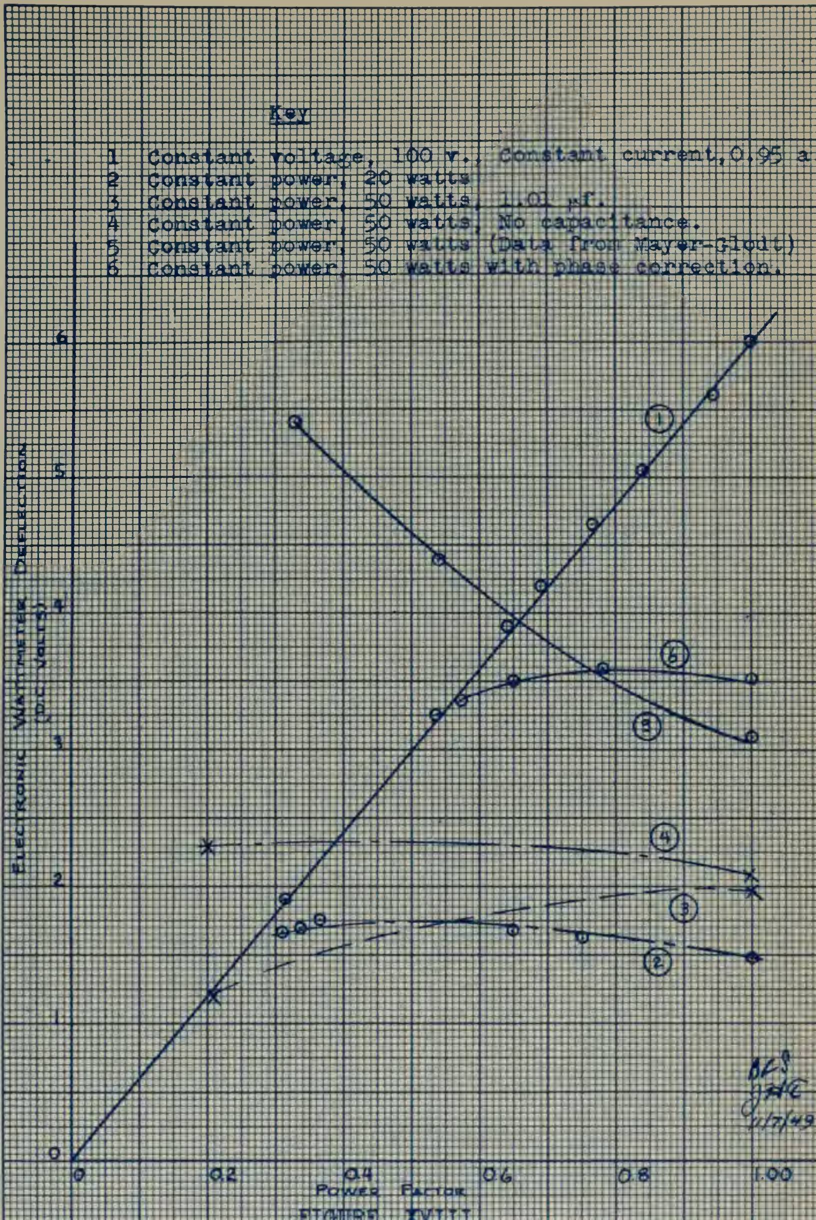


FIGURE XVIII

of this line can be varied at will in such a manner that the response over a limited range of power factor is nearly constant. A demonstration of this control is given by curves 3 and 4 in which the slope is shifted through the horizontal by a variation in the phase-shift introduced. (Note: curves 3 and 4 were taken at a later date than curves 1 and 2 and are not directly comparable due to various circuit changes made in the intervening period.)

Although not confirmed, it appeared that the phase-shift did not occur, except in part, in the transformers. No attempt was made to isolate the phase-shift since it could be corrected. It was noted, however, that the amount of correction necessary changed when the 6A8 tubes were interchanged or replaced.

Despite the fact that the results of Pierce were thus duplicated subsequent experiments made it apparent that D.C. drift of the circuit was excessive for the application intended. Extensive tests led the authors to the conclusion that the main cause of this drift was a change in tube characteristics.

Due to the drift difficulties encountered with Pierce's circuit, investigation of the El Said circuit[3] was confined to tests for drift. Tests on tube 954 used in the El Said circuit showed that it was also subject to drift to the extent that its use would render a wattmeter circuit inaccurate unless frequent recalibrations were made. Such recalibrations were considered to be impractical.

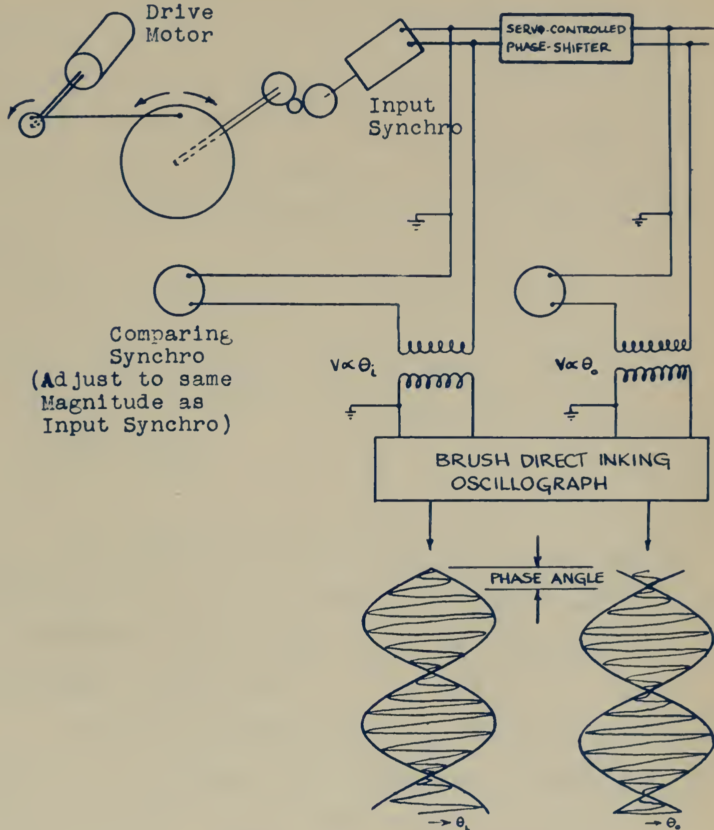
B. A Proposed Procedure for Making a Frequency-Response Test on the Phase-Sensitive Servomechanism.

Since the observed performance of the system indicates that there is an appreciable lag in the phase-sensitive rectifier, any frequency-response tests upon the system should include this unit. As the proposed change in gear ratio to 1000 will make the closed-loop system sufficiently stable for frequency-response tests (or will make them unnecessary), the method proposed below is for a closed-loop test. However, it can be easily adapted to an open-loop test, if desired.

In order to obtain a sinusoidal input signal, a variable-speed drive is utilized to oscillate the input synchro through an eccentric drive. The speed of the drive should be adjustable to one cycle per second, or less, since the resonant frequency is about ten radians per second.

The input and output voltages are bucked against comparing voltages of the same magnitude to obtain a signal which is proportional in magnitude to the two-phase angles. See Figure XIX.

Since these voltages are not grounded, they are sent through isolating transformers to permit grounding one side of the input to the Brush recorder amplifiers.



PROPOSED FREQUENCY-RESPONSE

TEST ARRANGEMENT

FIGURE XIX

C. A Possible Method of Using An A.C. Two-Phase Drive Motor

As mentioned in the body of this report, the system presently employed on the Network Analyzer to measure phase-angle is to null the unknown voltage against the output from a calibrated phase-shifter. The accuracy and simplicity of the method prompts the proposal for employing the null procedure in a phase-sensitive servomechanism as indicated in Figure XX.

From Figure XX, for $\theta_1 = 0$

$$\frac{T_L}{r\theta_o} = \frac{T_L}{r\varepsilon} = \frac{1}{r(J + \frac{J_L}{r^2})s^2} + K_1K_T + rKbsK_T$$

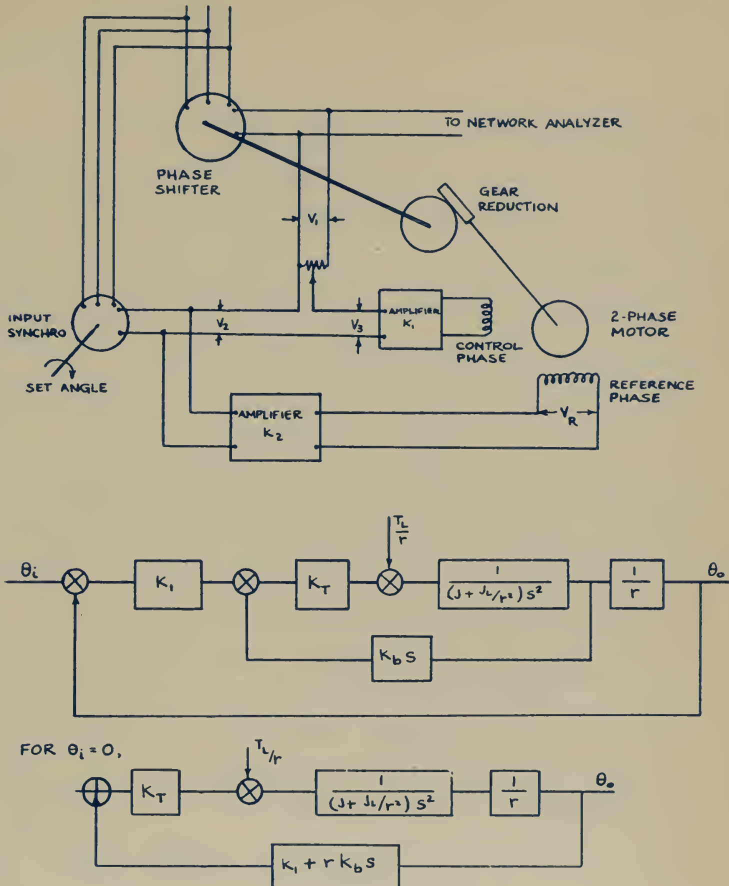
$$\therefore \frac{T_L}{\varepsilon_{ss}} = rK_1K_T \qquad (6)$$

The voltage of the phase-shifter is tapped off the potentiometer to be equal to the input synchro voltage. When the two voltages are in phase, the voltage applied to the control phase will be zero.

If the two voltages move out of phase, a resultant voltage, V_3, is amplified and applied to the control phase. Since V_3 is at 90^o to V_r, a motor torque will result. If V_1 and V_2 should differ in magnitude, a voltage, V_3, will be applied when the two voltages are in phase, but the motor torque at standstill will be zero.

From Equation (6), the over-all torque constant, T_L/ε_{ss}, will be proportional to r, the gear ratio. The

allowable gain for stability will, however, be inversely proportional to r. By using a sufficiently large value of gear ratio, it should be possible to make the system sufficiently "stiff." A.C. amplifiers can be used throughout the system, thus minimizing problems of drift.



PROPOSED CIRCUIT USING A TWO-PHASE MOTOR

FIGURE XX

D. The following brief analysis is included to demonstrate the effects of unbalance in the phase-sensitive rectifier circuit:

As shown in the body of this report, zero direct current output of a correctly balanced phase-sensitive rectifier requires a 90^{o}-phase difference between the input voltages and is independent of the voltage magnitudes within the limits of the diodes used. If, however, transformer T_1, of Figure V, is not accurately center-tapped, the phase difference required for zero output voltage (D.C.) becomes a function of the magnitude of unbalance.

Assume in Figure VI, that $|\overline{bc}|$ is made larger than $|\overline{ab}|$ by an amount $\Delta\overline{bc}$ so that $|\overline{bc}| = |\overline{ab}| + |\Delta\overline{bc}|$. For zero D.C. output, $|V_1| = |V_2|$

$$|V_1| = j|\overline{ab}| + j|\Delta bc| + \overline{bd}\cos\theta + j|\overline{bd}|\sin\theta$$

$$|V_2| = -j|\overline{ab}| + j\ |bd|\sin\theta + \overline{bd}\cos\theta$$

$$\therefore \left| j|\overline{ab}| + j\Delta\overline{bc} + j\overline{bd}\sin\theta \right| = \left| -j\overline{ab} + j\overline{bd}\sin\theta \right|$$

$$\overline{ab} + \overline{\Delta bc} + \overline{bd}\sin\theta = \overline{ab} - \overline{bd}\sin\theta$$

$$\sin\theta = \left| \frac{\Delta\overline{bc}}{2\overline{bd}} \right|$$

For $\overline{bd}$ constant in magnitude, θ will vary directly as the magnitude of the input voltage $|\overline{bc}|$ up to a limit set by the amount of unbalance. Unbalanced tube characteristics would contribute a similar error.

E. Sample Calculation

1. To determine amplifier requirements:

Although some of the parameters in the diagram (Figure IX) are not susceptible to ready measurement, and no attempt was made to define the transfer function of the phase-shifter itself, the equations derived from this analysis serve to indicate the influence of certain parameters on system performance.

To evaluate the torque constant, K_t,

$$K_t = \frac{T_L}{\mathcal{E}_{\text{steady state}}}$$

which will allow the prediction of the error caused by a load torque, consider θ_1 constant:

Then

$$\frac{T_L}{r\theta_c} = \frac{1}{r(J_M + \frac{J_L}{r^2})s^2 + Fs} + \frac{K_d K_1 \mu_2 K_M}{r_p + r_f + sL_f}$$

Since $\theta_o = \mathcal{E}$, for steady-load torque,

$$\frac{T_L}{r\mathcal{E}_{ss}} = \frac{K_d K_1 \mu_2 K_M}{r_p + r_f} \qquad \therefore K_T = \frac{T_L}{\mathcal{E}_{ss}} = \frac{rK_d K_1 \mu_2 K_M}{r_p + r_f} \qquad (5)$$

For the assumed load torque of 2 ft-lbs. and a torque error of 0.5°,

$$K_t = \frac{2 \text{ ft-lbs.}}{0.5^\circ} = \frac{384 \text{ in-oz.}}{0.5^\circ} = 768 \text{ in-oz. per degree.} \qquad (6)$$

Assuming input voltages to the demodulator are approximately 80 volts rms from the phase-shifter and 40 volts rms. from the synchro-transformer: Then from Equation (3),

$$K_d = \frac{|V_1| - |V_2|}{\theta} = \frac{2\overline{bc}\,\overline{bd}}{\sqrt{\overline{bd}^2 + \overline{bc}^2}} = \frac{2(20)40}{\sqrt{2000}} = 35.8 \text{ volts/radian}$$

$$= 0.625 \text{ volts/degree}$$

(Note: This calculation is in error by the modification introduced by the filter circuit but it indicates primarily the order of magnitude expected.)

Available information on the motor indicated that a torque constant, K_m, of about 77 in-oz. per ampere could be expected.

Substituting these values in Equations (5) and (6), an estimate of the amplifier gain requirements was made:

$$K_d = 0.6 \text{ volt/degree}$$

$$K_m = 77 \text{ in-oz./amp.}$$

$$r = 100$$

$$\frac{K_1 \mu_2}{r_p + r_f} = \frac{T_L}{\xi_{ss} r K_d K_m} = \frac{768}{100(77)(0.6)} = 0.167 \text{ amp./volt.}$$

Assuming, $\frac{\mu_2}{r_p + r_f} = gm = 5000$ mhos (for use of 6L6's in power amplifier)

$$K_1 = \frac{0.167}{5000 \times 10^{-6}} = 33.4 \text{ volts/volt.}$$

Accordingly, a 6SL7, having a μ of 40 was used.

2. <u>Calculations for the constant-current source</u>

(See Figure X.)

Motor Data:

Rated Armature Current = 0.5 amp.

Armature Resistance = R_a = 35

Rated Armature Voltage = E_a = 110 v.

6L6 Beam Power Amplifier Tube Data:

Allowable Plate Dissipation = 19 watts

Allowable Screen Dissipation = 2.5 watts

<u>At standstill</u>, motor voltage = $I_a R_a$ = (.5)(35) = 17.5 v.

From tube characteristics, plate voltage = 152 volts.

Plate current per tube = 0.50/6 = 0.083 amps.

Plate dissipation = (152)(0.083) = 12.6 watts per tube

<u>At full speed</u>, motor voltage = 110 volts

Tube plate voltage = 70 volts

Plate current per tube = 0.075 amps.

Plate dissipation = (70)(0.075) = 5.25 watts

Note: (1) It was decided to operate near the "knee" of the tube characteristics in order that excessive voltages would not be applied to the motor under low-torque conditions.

(2) Tube data available was not sufficient to allow accurate calculation of screen dissipation. Since rough calculations indicated that screen grid dissipation would be excessive for the use of 4 or 5 tubes under the desired conditions, 6 tubes

were used. Experimental results might indicate that 3, or even 4, tubes could be satisfactorily used.

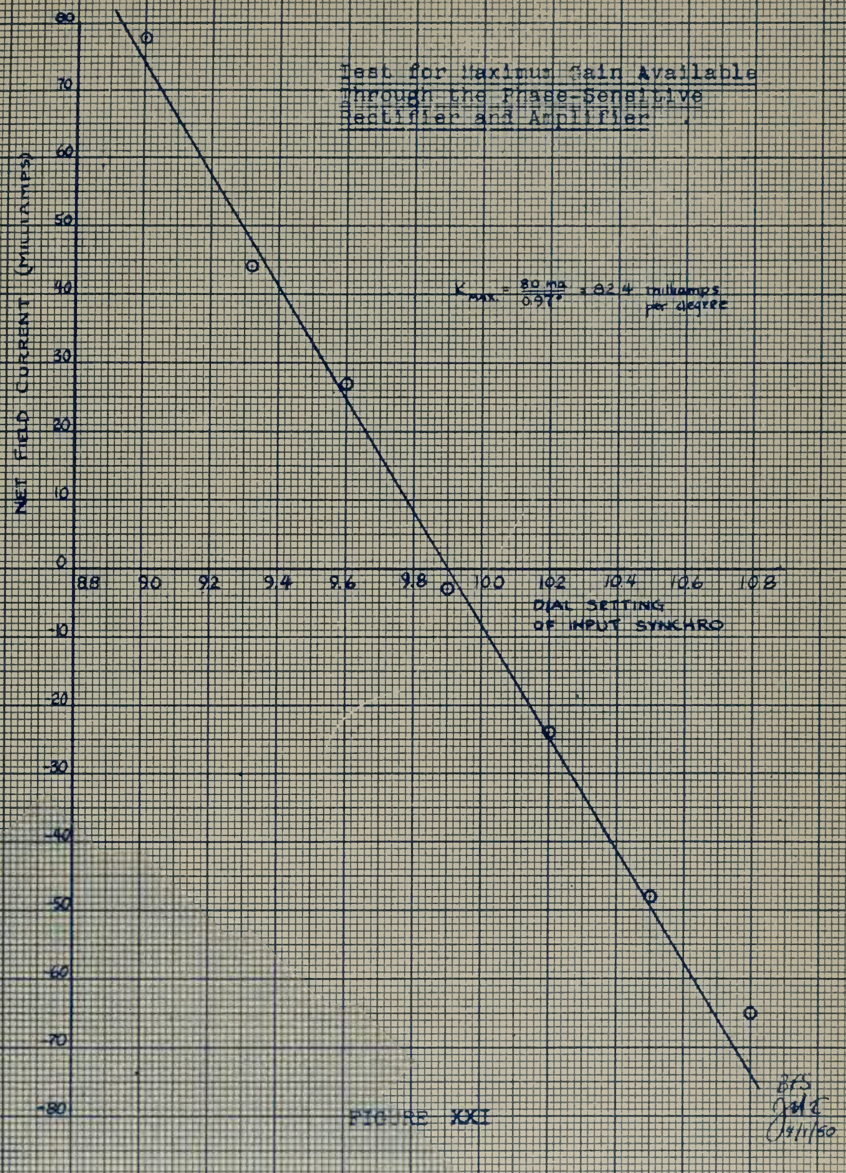
([illegible]) The entire circuit was designed with the intention of using the 220-volt D.C. mains and thus eliminating the need for an electronic power supply.

F. Typical Data

Table II

Calibration of Input Synchro-Transformer

Dial Setting	Measured Phase of Synchro Voltage	Error
0°	0.1°	0.1°
36	32.8	-3.2
72	71.6	-0.4
108	105.0	-3.0
144	140.0	-4.0
180	178.6	-1.[illegible]
216	211.9	-4.1
252	248.5	-3.5
288	285.3	-2.7
324	321.0	-3.0
360	362.9	2.9



Test for Maximum Gain Available Through the Phase-Sensitive Rectifier and Amplifier
NET FIELD CURRENT (MILLIAMPS)
DIAL SETTING OF INPUT SYNCHRO
$K_{MAX} = \frac{80 \text{ ma}}{0.97°} = 82.4$ milliamps per degree
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
8.8
9.0
9.2
9.4
9.6
9.8
10.0
10.2
10.4
10.6
10.8
BFS
4/1/50

FIGURE XXI

Table III

Drift Run:

Phase-shifter output voltage constant, 76 v.

Phase-shifter load constant, 400 ma.

Input synchro setting constant

Time	Measured phase angle of phase-shifter voltage in degrees.	Drift (Degrees)
1050	324.4°	-
1102	324.2	+0.2°
1108	324.9	-0.5
1114	324.3	+0.1
1118	324.6	-0.2
1133	324.8	-0.4
1141	324.7	-0.3
1146	324.2	+0.2
1150	324.6	-0.2

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